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THE STRENGTH OF THE EARTH'S CRUST

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PART V. THE DEPTH OF MASSES PRODUCING GRAVITY ANOMALIES AND DEFLECTION RESIDUALS

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INTRODUCTION AND SUMMARY

The fact that the observed deflections of the vertical are on the average only one-tenth as large as the computed effects of the topographic relief, computed on the assumption of uniform density throughout each earth shell, shows that the densities of the crust are not uniform, but in a broad way are balanced against the relief. This is the proof that a condition prevails of approximate regional isostasy. As the relief of the globe is highly variable, the densities in the lithosphere are therefore within certain limits also highly variable. But on the other hand, the existence of gravity anomalies and deflection residuals indicates that the variations in density are not completely in accord with the demands of the hypothesis which postulates local compensation of the topography uniformly distributed to a uniform depth, nor apparently with any other simple hypothesis. These quantities measure the differences between the hypothesis and the facts of nature. Let the density variations beyond those required to balance the topography vertically above them be called the outstanding excesses or defects of density and the masses which they represent be called the outstanding masses.

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[1] Section B of Part V, on the applications of the criteria to determine the limiting depths, forms, and masses of the excesses and defects of density, will be published in the following number of this *Journal*.

It is fundamental to the problems of the strength of the crust, and a system of geologic dynamics in accord with that strength, to determine the depth, form, and weight of these outstanding masses. Do they belong to the centrosphere or to the lithosphere? As Gilbert has noted, if they are to be referred to the centrosphere they do not imply any imperfection of isostasy nor any competence for stress within the crust. Or, if they exist in the lithosphere, the zone of compensation, but are balanced vertically in the same column by other masses of opposite sign, this arrangement will produce local strains within the crust but not tend to flex the crust as a whole. Neither in this case, therefore, would they measure departures from perfect isostasy. As following questions, are the imperfections of isostasy small and local, and the residuals and anomalies the summation of many scattered effects? Or, on the contrary, are there notable regional departures from the conditions of solid flotation which measure a very appreciable rigidity of the crust? If so, to what extent are these regional outstanding masses related to the mountains, valleys, and deltas in process of evolution under the present cycle of surface activities; producing a progressive unbalancing possibly being slowly restored toward balance by a viscous undertow? To what extent are the departures from isostasy due to variable composition and density of igneous intrusions dating back to earlier geologic ages, perhaps never in isostatic balance, and supported permanently and rigidly by the strength of the crust?

These problems have been studied in the last three parts of this article by means of the evidence presented by Hayford and Bowie, but that evidence has not been used to solve the depth of the outstanding masses. Yet it is seen that all of the aspects just enumerated are bound up in that factor. It is especially this problem of the depth and the consequent areal extent and mass of the units which produce the residuals of the deflection and gravity observations which is attacked in this part. It is necessary for this investigation to enter into a study of the complex relations between the anomalies and residuals which depend upon the depth and form of masses. It is a subject upon which, so far as the writer is aware, but little has been done, so that about half of this chapter, published as Section A, consists of a study of these relations preliminary to their application.

For facility of mathematical treatment the individual outstanding masses must be regarded as equivalent to spheres, spheroids, or cylindrical disks, either as units or as aggregates. If only the epicenter (the point on the surface vertically above the center) of the disturbing mass is determinable and the deflections at two or three points on one side of it, then the mass may be most simply interpreted as a sphere; since the mass of a sphere acts as if concentrated at its center. With fuller observations a close approximation to the depth of the mass and a less close approximation to its form and density may be made. The first problem then is to determine the epicenter of the outstanding mass and its depth, using for this purpose the nature of the anomalies and residuals. For a sphere beneath a plane surface it is shown that the value of the maximum gravity anomaly at the surface is 2.6 times the value of the maximum deflection residual, both being measured in the same units of force. The former occurs vertically over the abnormal spherical mass, that is, at the epicenter. The latter occurs at a horizontal distance from the epicenter equal to 70 per cent of the vertical depth to the center of mass. Oblate spheroids and broad cylindrical disks with vertical axes and the same depth of center as the sphere give maximum deflections at greater distances from the epicenter. The curves of the deflection force for these forms, especially the spheroidal forms, resemble somewhat closely those given by deeper spheres of greater mass. If the outstanding masses are in reality horizontally extended the interpretation of the deflection residuals as due to spherical masses assigns to their centers in consequence too great a depth. If, however, the masses have the forms of vertical prolate spheroids or vertical elongate cylinders, the interpretation as spheres will give too shallow a depth. The ratios of the maximum anomalies to the maximum deflections constitute a criterion to show whether the masses depart from spheres by the spreading-out of their substance in a horizontal plane or along a vertical axis.

In Section B the outstanding masses are shown in some cases to be horizontally extended in form and this is thought for the larger masses to be a rather general relationship; that is, the vertical thickness is much less than the length or breadth. Consequently the interpretation as spheres gives maximum limits to the depth.

A general inspection of the geodetic data as well as a detailed study of a certain test region shows that the smaller disturbing masses have their centers in the outer third of the zone of compensation; that is, within 40 km. of the surface. This result is to be expected, since similar small masses at greater depth would not exert a notable effect because their gravitative force varies inversely with the square of the distance. But evidence of more significance is found in regard to the larger centers of outstanding mass not related to topography. These also are found, in so far as they have been investigated, to be situated in the outer third of the zone of compensation. Yet these masses are capable of showing notable effects to distances of from 100 to 150 km. If they were situated at any depth within the zone of compensation they would, therefore, betray both their existence and their depth. The greater departures from isostasy appear, therefore, to be really absent from the deeper parts of the lithosphere.

Centrospheric heterogeneity, if present, would require greater masses in order to show surface effects. But no such effects are noted. In so far as they may be existent, they are largely masked by the more important attractions of superficial masses and hidden by the indeterminate nature of much of the present data. Therefore centrospheric heterogeneity is not a hypothesis which can be used to account for the apparent departures from isostasy. It can be at most only a very secondary factor.

In the last topic of Section B is discussed the relation of the depth of outstanding masses to the various hypotheses regarding the distribution of compensation. The hypothesis of local compensation, as is perceived to a certain extent by those who have used it, is in error in supposing that variations in density correspond to every topographic feature and extend uniformly to the bottom of the zone. But these errors, whether they be small or great, are so spread out in depth and their centers of attraction are consequently so far removed from the surface that they have little effect on the geodetic observations. Especially is this true in comparison with those large and concentrated outstanding masses due to batholithic invasion or other causes which are found to exist at moderate depths in the outer crust. The reasons then why the deflection residuals and gravity anomalies appear to show so little relation to local surface relief and larger physiographic provinces are threefold: in part because of a regional compensation; in part because the hypothesis of local compensation as here shown masks the error contained in the assumption of perfect and local isostasy; in part because for many regions the ancient heterogeneities of mass hidden within the crust seem in reality to be greater than the heterogeneities of mass visible at the surface in topographic form and created by present gradational actions.

The results of this chapter converge with the lines of evidence previously considered and confirm them in showing considerable defects from isostasy for areas which are 100 km. or more in radius. This confirmation is to be expected, since it would be indeed remarkable if a crust, competent to carry such loads as the *geologic* evidence from erosion and sedimentation shows to be imposed, should give *geodetic* evidence of fairly local and nearly perfect adjustment between the topographic forms, developed by present external processes, and the variations in density imposed by past internal forces.

SECTION A

DEVELOPMENT OF CRITERIA FOR SPHEROIDAL MASSES

Separation of lithospheric from centrospheric outstanding masses.—Let the zone of compensation be regarded as the boundary of the lithosphere. At its bottom consider to exist a zone in which that lateral flowage takes place which is necessary for movements of isostatic readjustment and the maintenance through geologic time of a condition of more or less complete isostasy. Below it is the inner and more rigid core of the earth, the centrosphere. Let those excesses or defects of density above the zone of isostatic flow which are not in accord with the isostatic compensation of the topography be designated for convenience as lithospheric outstanding masses. Let all heterogeneities of density within any earth shell below the zone of isostatic flow be called centrospheric outstanding masses.

In his recent paper on the "Interpretation of Anomalies of Gravity,"[1] Gilbert calls attention to the fact that if abnormalities of density exist below the zone of compensation they will produce anomalies of gravity without these signifying real departures from isostasy. This is a very necessary addition to the theory of the cause of gravity anomalies and deflection residuals. As a test, Gilbert has calculated the influence of a right cylinder with vertical axis, of density ±0.025, with height and radius each equal to 122 km., whose upper surface is at a depth of 122 km., thus reaching up to the bottom of the zone of compensation as given by Solution H. Such a cylinder would give a maximum anomaly of ±0.023 dyne at the epicenter, a quantity of the same order of magnitude as the mean anomaly for the United States, 0.018 or 0.020 dyne.

In the application of this test to the earth it would appear, however, that two things should be noted. First, to account for the *mean* anomaly of 0.020 dyne the centrospheric masses would have to be several times as great as this cylinder, even for this depth of 122 km. to the top surface, since the maximum value of the anomaly occurs at the epicenter of the mass, and for a cylinder of the form postulated falls off rapidly with increasing horizontal distance from the epicenter. Second, the test mass has been taken as contiguous to the zone of compensation above and with that limited depth given by the hypothesis of uniform compensation. This gives it greater effect according to the law of inverse squares, but postulates either an indefinitely thin zone of isostatic flow at the bottom of the zone of compensation or a capacity in a thicker zone of weakness to maintain within itself heterogeneities of density similar to those of the lithosphere above and the centrosphere beneath. It is thought by the present writer that a more probable presumption is that the centrospheric heterogeneities which may exist are distinctly deeper than 122 km. and separated from the lithospheric outstanding masses by a thick zone which yields to broad inequalities of pressure either upon it or within it and therefore is incapable of maintaining notable inequalities of mass in this shell.

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[1] *Professional Paper 85C, U.S. Geol. Survey*, 1913, pp. 35, 36.

The reasons for this preliminary hypothesis are briefly as follows: The depth of compensation seems to be variable and to extend in some regions to as much as 300 km., even under the assumption of compensation uniformly distributed and complete at the bottom. Under a more natural assumption that isostatic compensation gradually disappears, those heterogeneities of density which give isostatic compensation would gradually diminish with depth and this diminution would extend to a considerably greater depth than 122 km. If heterogeneities which act isostatically gradually disappear, the heterogeneities which can be borne in excess should also be expected to diminish.

As to the nature of the shell immediately below the zone of compensation, Schweydar has recently analyzed mathematically the results of the measurements of earth tides by means of the horizontal pendulum.

The calculations were designed to test the presence or absence of a viscous zone between an elastic crust and elastic interior. It is concluded that even a magma bed with a viscosity as high as that of sealing wax at house temperatures and a thickness of but 100 km. cannot be present. The assumption in best agreement with observations is that of the presence of a layer about 600 km. thick, slightly ductile (coefficient 10^{13} to 10^{14}), existing beneath an outer crust 120 km. thick.[1]

By postulating such a thick zone for isostatic flow, the viscous resistances are reduced and solid flow is made easier. It also is in conformity with the probability of a gradual change of physical state from the rigidity above into a less rigid and less stable tract and this in turn into a more rigid interior. Now if such a thick viscous zone is incapable of supporting over broad areas loads imposed by abnormalities of density above, it should also be incapable of supporting such horizontal inequalities of mass within it, provided these are sufficiently large. But in order to produce the same gravitative surface effects as more superficial masses, the heterogeneities of this zone of viscous flow would in fact have to be much larger. A cylinder of the dimensions postulated by Gilbert, if of negative density and adjacent to another at the same depth but of positive departure from the mean density, would tend to be underthrust by the latter, and the denser would in turn tend to be overflowed by the lighter.

For these reasons it is not to be expected that the same departures from those densities giving isostatic equilibrium which could exist in a rigid shell above would extend immediately below.

Influence of centrospheric heterogeneity.—To test the question of the influence of heterogeneity below the zone of compensation a sphere will be considered. First, one whose center is at a depth of 319 km., 0.05 of the radius of the earth. As a second test, the influence will be determined of a sphere whose center is at a depth of 637 km., 0.10 of the earth's radius. For considering the attraction of a mass at points on the surface other than at the epicenter it is more convenient to take the mass as having the form of a sphere rather than a cylinder, since the mass of the sphere acts in all directions as if concentrated at its center. This favors, furthermore, the accentuation of the effects upon the surface over what they would be if the outstanding mass had a stratiform extension.

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[1] Dr. Wilhelm Schweydar, "Untersuchungen über die Gezeiten der festen Erde und die hypothetische Magmaschicht," *Veröffentlichung des k.k. Preusz. geodät. Institutes,* Neue Folge No. 54, Leipzig, 1912 (B. G. Teubner).

The sphere having the same volume as a cylinder 122 km. in radius and 122 km. in depth will have a radius of 111 km. Let a radius of 100 km. and a density of ±0.025 be assumed as the dimensions and mass of a standard sphere in this deep zone. If the center of such a sphere is at a depth of 183 km., the same depth as the center of Gilbert's postulated cylinder, the anomaly at the epicenter will be 0.021 dyne, whereas the cylinder gave an anomaly of 0.023 dyne. They are therefore nearly equal in effect. If the center of the sphere is placed at a depth of 319 km., making the top at 219 km., the anomaly at the epicenter becomes 0.0068 dyne. Consequently the variation in density or volume of the sphere would have to become three times as great in order that its maximum anomaly should equal the mean observed anomaly. But as the average anomaly is not measured at the epicenter, and the maximum anomalies, occurring at the epicenters, are several times the observed mean anomalies, this figure would have to be still further multiplied. To account, therefore, for the magnitude of surface anomalies, the disturbing spheres, if with centers at a depth of 319 km. and if of 100 km. radius, would have to have abnormalities of densities ranging up to 0.25 in order of magnitude. If the centers of the spheres were at twice this depth the abnormalities, to produce the same effect, would have to be four times as great in mass. In a region of which there is no precise knowledge such variations of density might well occur. The problem must therefore be investigated by means of the gradients which would result in the gravity anomalies and deflection residuals and a comparison of these with the gradients actually observed and plotted.

Distribution of surface forces for centrospheric spheres.—For masses as deep as these the curvature of the earth becomes of importance, but the complications which it introduces into the analytic treatment have been avoided by means of a graphic solution.

In Fig. 8A, the anomaly is calculated for the epicenter. Then the gravitative force at any other point on the surface, such as that having a dip angle θ, can be determined by squaring the inverse ratio of distance. Multiplying the force at the epicenter by this

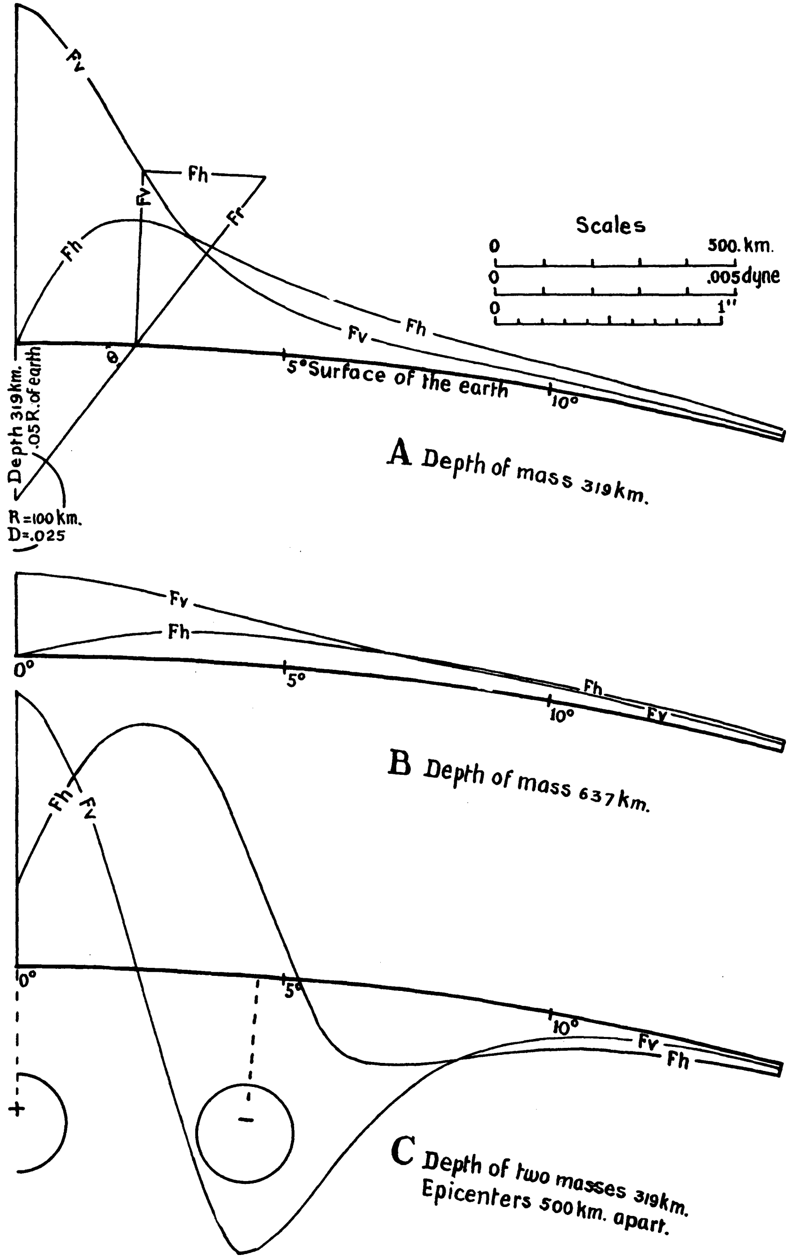

FIG. 8.—Curves showing values for the gravity anomalies *Fv*, and deflection residuals *Fh*, on the surface of the earth, for spherical masses situated at depths of 319 and 637 km., respectively. Ordinates measured at right angles to earth's surface.

factor gives the force at the second point acting in the direction of the radius of the attracting sphere. This value is laid off as *Fr* and then resolved into two components *Fv* and *Fh*, vertical and parallel respectively to the surface of the earth. The ratio of *Fv* to *Fh* is tan θ. With increasing distance from the epicenter θ becomes increasingly greater than it would be if the earth's surface were regarded as a plane. Therefore for distances of 5 to 10 degrees and more from the epicenter *Fv* begins to hold an appreciably higher ratio over *Fh* than it would if curvature were neglected. It is seen that $Fv=Fh$ for $\theta=45^\circ$. Nearer the epicenter *Fv* is in excess; at greater distances *Fh* is the greater. *Fv* is a maximum for $\theta=90^\circ$. *Fh* is a maximum for $\theta=55^\circ$ if curvature be neglected. For the earth's curvature and a depth of 319 km. to the center of mass, θ is a maximum for $53^\circ\pm$. The point giving this is at a distance from the epicenter of 0.75 the depth. The ratio of maximum *Fv* divided by maximum *Fh* is approximately 2.7. In Fig. 8C are shown the effects of two spheres of opposite sign but of equal mass. If these two spheres were superposed they would of course completely neutralize each other. Upon moving them horizontally apart to 1.5 times the depth, the maximum value of *Fh* becomes twice the value for a single sphere. This occurs halfway between them, and the value of *Fv* for this point is zero. The ratio of maximum *Fv* over maximum *Fh* becomes 1.1. Two equal masses of like sign would, on the contrary, give a maximum value of *Fv* and a zero value of *Fh* at a point halfway between them. These represent the extreme departures from the case of a single spherical disturbing mass. More distant masses show less overlapping of their fields of force and tend to have their individual effect upon a point between them neutralized by the larger number of masses acting from various directions. The values of *Fv* are much more under the control of the individual masses than are the values of *Fh*.

Returning to the single dominating mass of spherical form as shown in Fig. 8A, let the values of *Fv* and *Fh* be represented by ordinates as shown in the figure; then the surface representing the gravity anomalies, *Fv*, would be a dome of double curvature, like a craterless volcano; the surface representing the deflection force, *Fh*, would be in the form of a caldera or crater ring. According as the attracting mass departed in its nature from a sphere these shapes of the force surfaces would be modified, but the general character of volcanic cone and caldera would remain. Negative masses would have the forms reversed. If the disturbing masses are at distances apart which average several times their depth, then a relief map of the resultant forces would resemble a volcanic field with the volcanoes isolated from each other. If the centers are much closer, the relief map would come more to resemble those lunar craters which show all degrees of superposition upon older craters. The deeper the masses the broader the volcano-like curves of forces upon the surface, but the lower will be the relief, unless the disturbing spheres increase in mass with the square of the depth. Stratiform-like masses, such as oblate spheroids or cylindrical disks, will show less pronounced effects than the equivalent spheres and will simulate somewhat the effects of spheres at a greater depth. The attempt to apply these principles as criteria to the published data must be deferred until the influence of abnormal masses in the zone of compensation has been considered and also in somewhat more detail the influence of masses in other forms than spheres.

Influence of spheres within the zone of compensation.—The forces produced by spheres within this zone are more readily treated analytically, since it will be seen that the curvature of the earth may be neglected. Otherwise this topic is to a considerable degree an extension of the last. The unit mass which it is convenient to adopt for this discussion is that of a sphere whose radius is 50 km. and density 0.100, one-half the mass of the sphere previously considered. Its center is taken at a depth of 64 km., 0.01 of the earth's radius, and approximately at the middle of the zone of compensation as given by Solution H. This gives the greatest abnormality of mass in the middle of the zone of compensation and will approximate to the mean effect of an outstanding density distributed uniformly throughout that zone. At the epicenter the attraction is wholly effective in producing gravity anomaly and is measured by the formula

$$F=\frac{dc(\frac{4}{3}\pi R^3)}{D^2}$$

In this d=density, c=constant of gravitation, R=radius, D=depth. Solving this equation for the values chosen gives

$$F = .0853 \text{ dyne}$$

Take the earth's surface as a plane and any point on it as located by the dip angle θ, made by a line from the point to the center of the sphere of outstanding mass. Then the vertical component Fv and the horizontal component Fh are given by the following equations:

$$Fv \text{ (dynes)} = 0.0853 \sin^3 \theta$$
$$Fh \text{ (dynes)} = 0.0853 \sin^2 \theta \cos \theta$$

To convert Fh into seconds of arc divide by 0.00475 and

$$Fh \text{ (seconds)} = 17.94 \sin^2 \theta \cos \theta$$

The maximum value of Fh occurs for $\theta = 55°$ and is 0.0328 dyne or 6.9 seconds. The curves for Fv and Fh are shown in the unbroken lines of Figs. 9 and 10. They are seen to be quite close in character to the curves of Fig. 8. Changes in the mass or depth of the sphere will serve to change only the scales of forces and distances so that these curves may be adapted readily to apply to all spherical masses situated within the lithosphere.

Influence of sum of intersecting spheres approximately equivalent to spheroids.—The analysis of the gravitative forces which a sphere exerts upon points in an external plane serves as a starting-point for the consideration of the problem of the influence of those unit masses of excess or defect of density which exist in the crust. As a further step, any one mass may be considered as approximating in form either to some oblate or prolate spheroid or to some ellipsoid of three unequal axes. But the equations for the forces exerted by spheroids upon an external plane are complicated and laborious to solve. A sufficient approximation to the influence of a spheroid may be made, however, by employing several intersecting spheres which together give an approximation to the right quantity and distribution of mass. The influence of the composite mass is readily attained by summing up the curves given by the modifications for the several spheres.

In Fig. 9 the unbroken lines, as previously noted, are the curves of force due to the single unit sphere. The broken lines show the

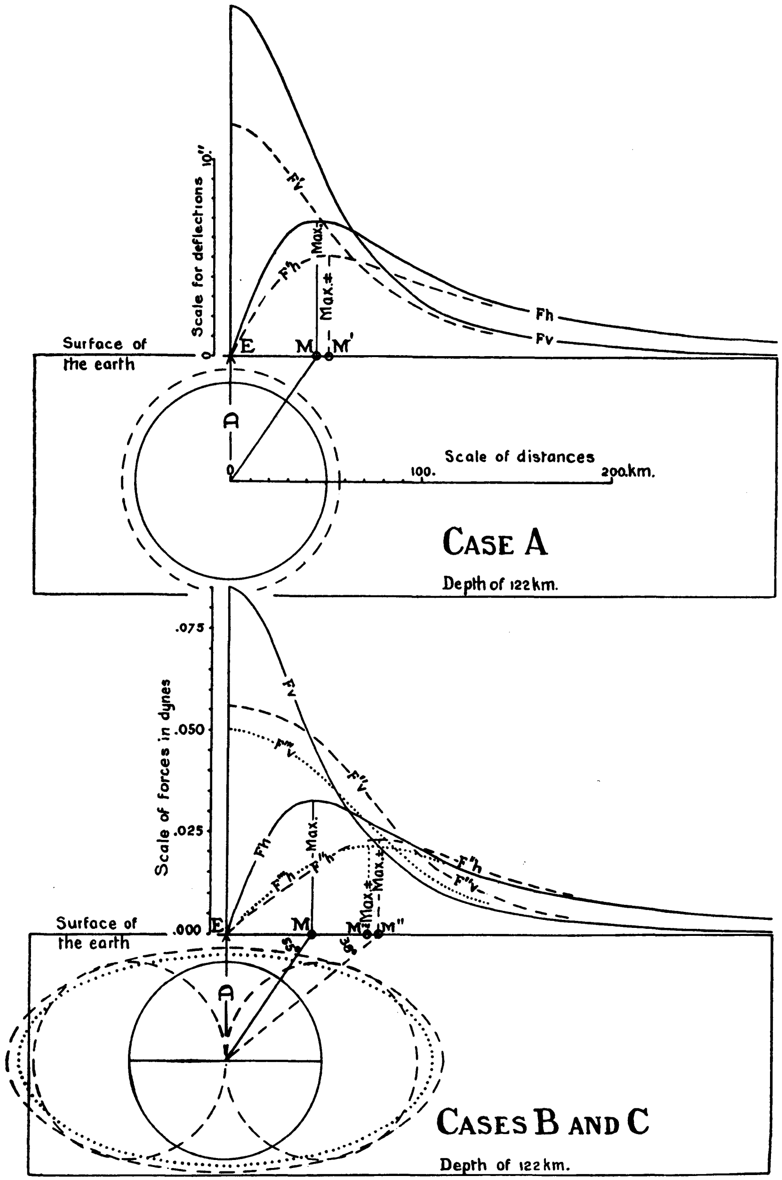

FIG. 9.—Various cases of horizontal and vertical components of the gravitative force due to a unit mass concentrated into a single sphere and expanded into several intersecting spheres. Cases A and B, three spheres in line. Case C, five spheres made by combining A and B and omitting one central sphere.

curves of force due to the same unit mass expanded into three spheres of the original dimensions and with centers 50 km. apart on a horizontal line. One-third of the mass is therefore in each sphere and the density of each is 0.033. In Case A the line joining the centers is at right angles to the vertical section plane. In Case B the line joining the centers lies in the section plane. The dotted lines show Case C. In this the unit mass is expanded into five spheres of the original size, each sphere possessing, therefore, one-fifth of a unit mass, and consequently a density of 0.020. In this case the five centers are arranged in a horizontal plane, the four outer spheres having their centers 50 km. from the center of the inner sphere.

The single sphere has a volume given by the formula $V=\frac{4}{3}\pi R^3$, in which $R=50$ km., and a density of 0.100. The three spheres have a volume therefore of $4\pi R^3$ and a density of 0.333. The three spheres make a solid of revolution whose semipolar axis is equal to $2R$, equatorial radius equal to R. Upon comparing this aggregate to a spheroid it is seen that the double density 0.667 of the intersecting portions compensates roughly for the two re-entrant zones on each side of the equator. To what regular spheroid does it approximate in its proportions? Let E be the equatorial radius of the spheroid and $2E$ the semipolar axis. The volume will be $\frac{8}{3}\pi E^3$, equal to the three spheres whose volume is $4\pi R^3$, or a single sphere of radius $1.44R$. Solving gives $E=1.14R$, $2E=2.28R$. The spheroid with these semiaxes is shown in broken lines in Fig. 9. This, then, is a spheroid which, if the density be taken as 0.033, is of exactly the same mass as the original unit sphere, or the three intersecting spheres, and which approximates in distribution of mass and in gravitative effect to these three spheres as shown in Fig. 9. The nature of the differences will be discussed later.

Case C shows five spheres of unit volume and of density 0.020 whose intersecting portions would consequently have densities of 0.040 and 0.060. In comparing the compound mass to an oblate spheroid these intersecting portions compensate roughly for the re-entrants between the spheres. The limiting dimensions of the whole in the directions of the three principal axes are *R* and 2*R*. Let it be required to find the value of the equatorial radius 2*P* and semipolar axis *P* of the oblate spheroid of equal volume in which the axes have these proportions. Then

$$\frac{16}{3}\pi P^3=\frac{20}{3}\pi R^3$$

$$P=1.08R$$

It is seen that the five spheres of density 0.020 have the same mass as a sphere whose radius is 1.71*R* and density 0.020; the same mass also as the unit sphere of density 0.100 and radius *R* and the oblate spheroid of density 0.020 and semipolar axis 1.08*R*, equatorial radius 2.16*R*. The vertical section of this spheriod is shown in dotted outline in Fig. 9. The distribution of mass and gravitative effect of the five spheres will be nearly the same as for such a spheroid. The nature of the differences, as in cases A and B, will be discussed later.

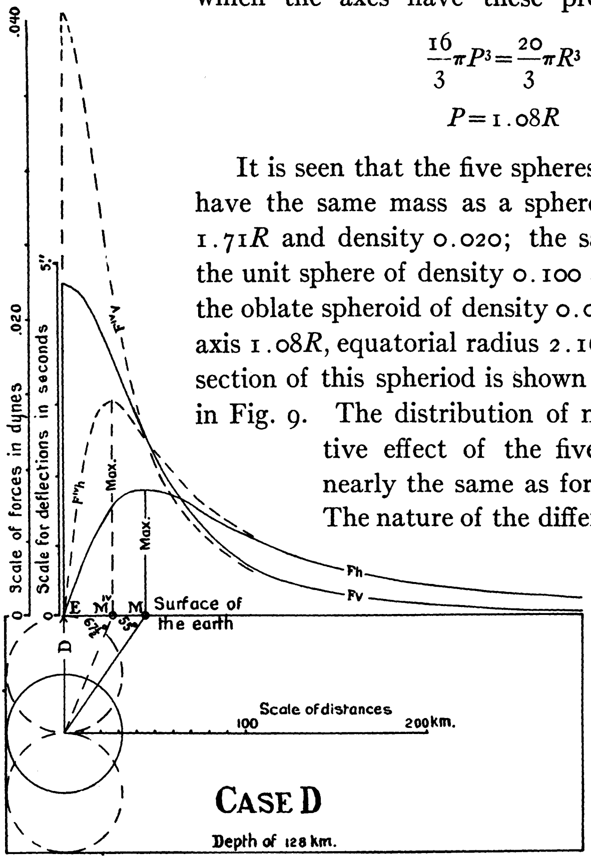

FIG. 10.—Horizontal and vertical components of the gravitative force due, first, to a sphere of 32 km. radius, density 0.100, depth to center 64 km., and second, to the same mass expanded into three such spheres with centers on a vertical line at depths of 32, 64, and 96 km.

The effect of the distribution of mass along a horizontal line and in a plane has been considered in cases A, B, C. There remains to be considered the effect of the distribution along a vertical line and in a vertical plane. Case D, Fig. 10, is given to show the effect of the distribution along a vertical line. The unbroken-line curves show the values of *Fv* and *Fh* for a sphere with density and depth corresponding to the unit sphere previously used, but with radius of 32 km. instead of 50 km. The mass is consequently but 26 per cent of that of the unit sphere. Let this be expanded into three intersecting spheres with centers 32 km. apart and arranged in a vertical line. The composite mass will then extend from the surface to a depth of 128 km. and correspond closely in effect to a prolate spheroid with vertical axis.

The resulting values of the components of the gravitative force for these four cases which are of importance for the development of criteria may be tabulated as follows:

TABLE XXV

	UNIT MASS		RATIO OF MAXIMUM *Fv* DIVIDED BY MAXIMUM *Fh*	VALUE OF θ FOR MAXIMUM *Fh*
	Maximum *Fv*	Maximum *Fh*		
Sphere	0.085	0.033	2.6	55°
Case A	0.056	0.024	2.3	51±
Case B	0.056	0.023	2.4	38±
Case C	0.050	0.022	2.4	41±
	0.26 UNIT MASS			
	Maximum *Fv*	Maximum *Fh*		
Sphere	0.023	0.009	2.6	55
Case D	0.041	0.015	2.8	67.5

To complete the series the curves should be drawn for five intersecting spheres arranged in a vertical plane analogous to Case V, first parallel and then at right angles to the plane of the section, making cases E and F, but the general character of the resulting curves may be inferred from the cases already given. Therefore, in order to abbreviate the discussion, an additional figure for cases E and F has been omitted.

It is seen from inspection of Figs. 9 and 10 and Table XXV that even for a constant mass and center of gravity the values of *Fv* and *Fh* change rapidly with the changing form of the mass. Upon the linear extension of a sphere into a form such as cases A and B the maximum value of *Fv* falls to two-thirds of its original value. For Case C it is still less. The ratio of the maximum value of *Fv* divided by the maximum value of *Fh* is also seen to change, but more slowly, decreasing for the spreading-out of the mass in a horizontal direction, increasing for a linear vertical extension. For the spheroids whose axes are in the relation of 1 to 2 and to which the spheres are equivalent in volume, the changes would be still more marked. This is because the duplications of mass due to the intersecting spheres are near the center and in Case C a top view would show considerable deficiencies of mass in the equatorial zone between the spheres and the spheroid. These intersecting spheres are consequently more effective in producing gravity anomaly and somewhat more effective in producing deflection of the vertical in the zone of maximum deflection. For cylindrical disks of the same mass and proportions as the spheroids the changes away from the values for a sphere would be still greater, since a greater proportion of the mass would be removed from the center to the edges of the body.

Distinctive effects of individual spheres and spheroids.—In the subjection of deflection measurements to the hypothesis of isostasy Hayford found it necessary to consider the effects of topography to a distance of 4,125 km. and in the determination of gravity anomalies the topography of the whole earth and its compensation were considered. To what extent then do distant masses affect the local residuals and vitiate any attempt to analyze the effects of local masses? In answer, it is seen that the effects of distant masses are negligible. It is the great topographic contrast of ocean basins and continental platforms, to a lesser extent the large variations of relief within these areas, which require their effects to be considered to such a great distance. But it is found that this larger relief of the crust is nine-tenths compensated and the outstanding masses so far as known do not show any marked segregation as to sign within the continental areas as opposed to the oceanic areas. Furthermore, the unit areas departing markedly from isostatic equilibrium are much smaller than these major segments of the crust. Therefore where it is the effect of the outstanding masses which is under consideration they are seen to be individually small in comparison with the greater relief features of the globe, and further, they mutually cancel their effects. The local outstanding masses are furthermore of the same order of magnitude as the more distant ones and therefore the effects of the distant masses sink to negligible quantities in comparison, in accordance with the law of inverse squares. The general agreement in the magnitude of the departures from isostasy, as shown by deflection residuals and gravity anomalies, shows, furthermore, that deep nucleal heterogeneities can have no large and broad regional effects, since such would affect the gravity measurements within a broad central circle to a greater degree than they would the deflections of the vertical. Therefore each region is seen to offer its local problems and the dominating centers of outstanding masses may be readily determined save where several such masses are contiguous, especially if of opposite sign. Let attention be given then to those features in the influence of outstanding masses which are indicative of the form and depth of the attracting mass.

Let the depth to the center of mass be D. Then it is seen that for ellipsoids near the surface in which the major axes are twice the minor axes the influence of form has mostly disappeared at horizontal distances from the epicenter or from $2D$ to $3D$, and at greater distances the curves become practically coincident. For greater departures from the spherical form the distances before the curves approach those given by a spherical mass are still greater. At these distances where the curves approach those of spheres, the effects of the form of the mass could not be distinguished, but the curves are so flat that neither could the effects of depth be readily evaluated. For instance, the curves of force at $4D$ to $6D$ for a sphere of mass M at depth D would be approximately the same as for a sphere of mass $4M$ at a depth of $2D$. Furthermore, at these distances from the epicenter the forces are so small in proportion to the maxima for the same mass that other outstanding masses would greatly change their value and prevent a correct analysis. Therefore, to be determinative, observations must be made at a number of points between the epicenter E and a distance not more than double that which at the point M gives the maximum value to Fh. It is seen that if a mass symmetrical about a vertical axis departs widely from the spherical form, this will be detected by noting the ratio of maximum Fv to maximum Fh, the latter being measured along any line radiating from the epicenter. If the mass is unsymmetrical about a vertical axis, observations must be made along at least two lines at right angles to each other.

A minimum number of observations will define an isolated outstanding mass, but if several masses have their fields of force notably overlapping, a larger number of observations becomes necessary in order to differentiate their effects. An inspection of Figs. 9 and 10 shows that the shape of the curve of *Fv* between the epicenter and a distance where it falls to one-tenth the maximum value has more distinctive relation to the form of the immediately adjacent mass than has the curve for *Fh*. But the available geodetic data supply less information regarding the gradients of the gravity anomalies than for the deflection residuals. The latter are given along a certain belt of triangulation stations, whereas the gravity stations are located at long distances apart. Furthermore, but few of the gravity stations coincide with deflection stations. The present analysis will therefore rest upon the data giving the curve for *Fh*. This curve is flat at the top, so that the data will readily give the approximate value of the maximum but will not determine closely its distance from the epicenter. The value of θ will, however, be determined ordinarily on two sides of the epicenter by means of the deflection residuals and the mean will give a more reliable figure than either alone. But according to the form of the mass within those limits shown in Figs. 9 and 10 the value of θ may range from 38° to $67\frac{1}{2}$°. If the abnormal mass is assumed to have a spherical form, its center will lie at an angle of 55° below the maximum value of *Fh* and at a depth 1.4 the distance to the epicenter. The error in locating the points of epicenter and maximum *Fh* may cause the estimate of depth to be in error 20 per cent and yet this figure will show definitely whether the sphere lies within the zone of compensation or in the centrosphere. If the mass, however, is in reality a horizontally elongate mass, the change in the distance *EM* from the epicenter to the point of maximum *Fh* in two directions at right angles to the epicenter will show that fact. A check on the form of the mass may be obtained if the value of the deflection curve is known with fair accuracy to a distance from the epicenter of three times the distance of the maximum. Let the distance to the point of maximum value be *M*. The ratios of the value of *Fh* at *M* to the value at 3*M* are given below as measured from the curves.

RATIOS OF *Fh* AT *M* TO *Fh* AT 3*M*

Sphere	2.4
Case A	2.1
Case B	4.3
Case C	3.4
Case D	2.0

It is seen that, from the difficulty in the precise location of *M* and hence the difficulty in locating a point as at 2*M* or 3*M*, and furthermore the probability that other masses may influence to a degree not readily determinable the value at 3*M*, this test cannot ordinarily be determinative. However, for spheroids with polar axes vertical, markedly prolate masses give a ratio distinctly smaller than for a sphere, the curve of deflections falling off more abruptly; markedly oblate masses, on the other hand, give a ratio distinctly larger than that for a sphere, the curve of deflection beyond the point of maximum being flatter.

If the outstanding mass approximates to an oblate spheroid with polar axis vertical, then the assumption of a spherical nature will locate the center of mass too deep and imply a greater mass than really exists. If, on the contrary, the form of the outstanding mass approaches the form of a vertical prolate spheroid, the interpretation of the deflections as caused by a sphere would locate the center of figure too high and give it too small a mass.

Suppose the curves for *Fh* shown in cases C and D have their maxima well determined in position and in magnitude, but that the values of the curves at distances two or three times beyond are not accurately known. Let these maxima be interpreted as produced by spheres. The depth and masses of the spheres which give these maximum deflections will be too great by the amounts shown in the following tabulation (Table XXVI, p. 462).

Where the data are sufficiently complete the form and depth of mass may both be determined, though a high precision is not to be expected. But in most cases with the present geodetic data the form of the mass will not be determinable and all that can be done is to interpret the deflections as produced by spherical masses. What then are the geological suggestions as to whether vertical prolate or oblate forms may be expected to characterize the larger outstanding masses? In the one case the error of interpretation will be to make the masses appear too small and shallow; in the other case, to make them appear too great and deep.

TABLE XXVI

Errors Due to Interpretation as Spheres of Unit Masses at Depth *D*

Form	Data		Assumption	Resulting Interpretation	
	Fh Max.	*EM.*	θ	Depth to Center	Mass
Case C (True)	4″.5	74 km.	41°	1.0*D*	1.0*M*
Interpretation as a sphere	4.5	74 km.	55	1.6*D*	1.8*M*
Case D (True)	3.05	27 km.	67.5	1.0*D*	1.0*M*
Interpretation as a sphere	3.05	27 km.	55	0.6*D*	0.6*M*

Stocks, and especially volcanic pipes, approach in form to vertical cylinders, but these are merely connecting structures. On the other hand, mountain ranges and geosynclines, although linearly extended, are of breadth which is great in comparison with the depth of excess or defect of density. Laccoliths and regional extrusions are also broad in comparison with depth. The relations as regards the great intrusive masses are not so clear, but erosion exposes batholiths over progressively greater areas; and whole provinces which exhibit regional metamorphism give suggestions that they are underlain by widespread igneous bodies. The hydrostatics of the magmas and their differentiation into masses of unlike density would also give tendencies to layers and horizontal extensions of the larger masses of abnormal density. These would depart, then, from the form of spherical masses in the direction of oblate spheroids with their equators in a horizontal plane. Narrower belts of disturbance like that which passes through Washington, D.C., may, on the other hand, tend to have the form of vertical plates. Therefore in none but the smaller and connecting structures are there geological suggestions of vertical prolate form.

The summation of this discussion shows that if in the first assumption as to the form of abnormal masses they be taken as spheres, then the determination of the depth of the center of mass by means of the curve for *Fh* and the location of the point of maximum value is more likely to overestimate than underestimate the depths and masses. As the object of this investigation is especially to find the depth of masses and to test the hypothesis of centrospheric heterogeneity as a cause of deflection residuals and gravity anomalies, it is desirable to have the error of interpretation in the direction of indicating a depth too great rather than too small. Therefore the initial assumption that the outstanding masses are spheres is justified by the geologic probabilities and is found in Section B to be justified by the geodetic evidence. The next topic will therefore develop further the subject of the interpretation as spheres with the view to utilizing the geodetic data.

Depths of spheres whose epicenters are not on the line of traverse.— If the primary purpose of a geodetic investigation were the determination of the location of the epicenters of abnormal masses and then the measurement of their form, size, and depth, a series of gravity and deflection measurements could be made in a line passing above the mass and near the epicenter. The preceding discussion would then directly apply. In only a few localities, however, will a line of triangulation stations, located in connection with the measurement of the earth's surface, pass approximately over the center of a large outstanding mass. How then, from the locations and values of the deflection force along any linear belt of measurements, shall the location of the epicenter and depth to the center of an abnormal mass to one side of the line of traverse be determined?

In Fig. 11 is developed a method for the solution of this problem. In accordance with the preceding discussion and the reconnaissance nature of a first investigation, let it be assumed that isolated abnormal masses approach a spherical form; that is, that a mass may be regarded as concentrated at a point. Take the center of the mass as the center of co-ordinates and the axis *X–X* as lying parallel to the line of traverse. The epicenter is at *E*. Then the vertical distance from the center to the epicenter is *D*. The

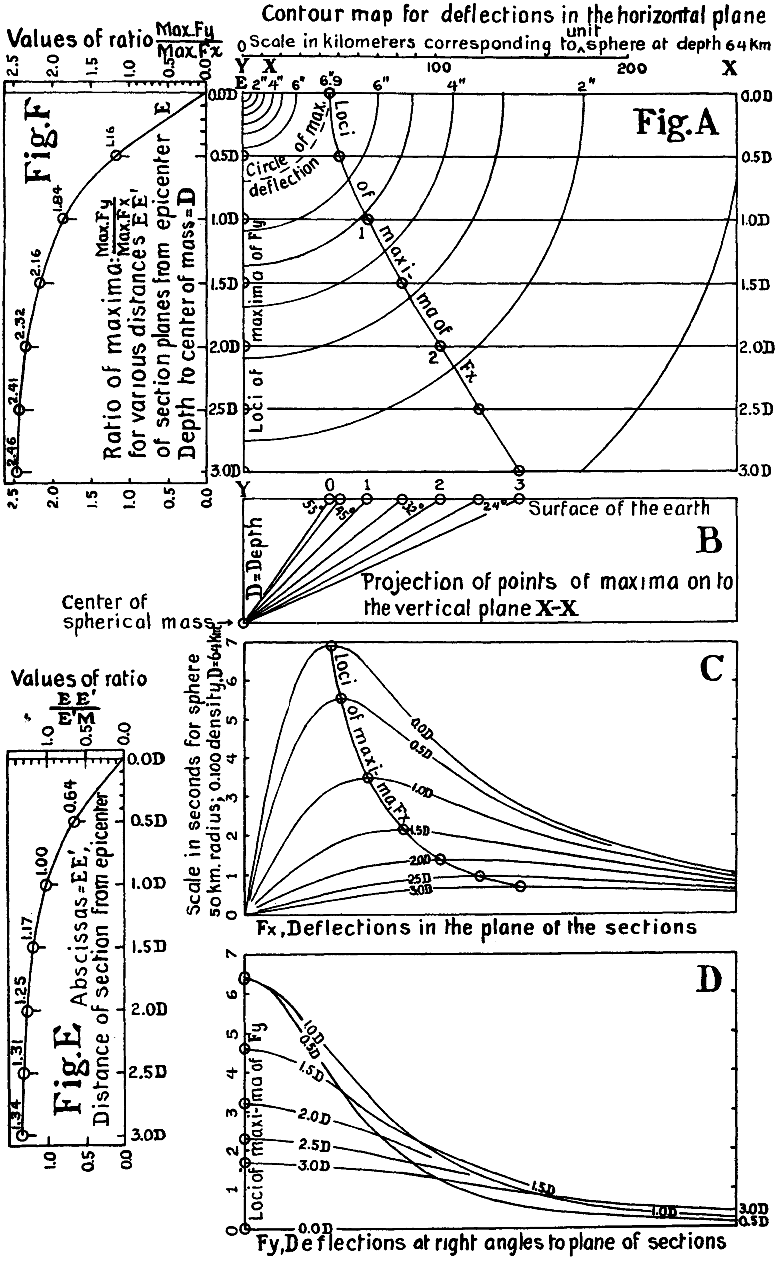

FIG. 11.—Values of the components of *Fh* on traverse lines which pass at various distances *EE'* from the epicenter *E* of a spherical mass at depth *D*; *EE'* being measured in terms of *D*.

distance *EE′* from epicenter to the traverse line is y. Then the co-ordinates of any point on the surface are x, y, and D. At any such point the component in the horizontal plane of the gravitative force due to the outstanding mass is Fh. But Fh is directed radially from all points to the epicenter. Fh, consequently, is subdivided into two components at right angles. Let that acting in the line of the section be called Fx, that at right angles be called Fy, parallel respectively to the X and Y axes.

The value of Fx for any point is given by the equation

$$Fx = dc\frac{4}{3}\pi R^3 \frac{x}{(x^2+y^2+D^2)^{\frac{3}{2}}}$$

and the value of Fy is given by the equation

$$Fy = dc\frac{4}{3}\pi R^3 \frac{y}{(x^2+y^2+D^2)^{\frac{3}{2}}}$$

Fig. 11 shows the results of the solution of these equations. In Fig. 11A is drawn a contour map of the deflection force produced by the unit sphere. The deflection force Fh at any point is measured by the contour map and is directed toward the epicenter. The lines of equal deflection are seen to be circles with center at E. They show in plan the values which were shown in section by the full lines for Fh in Figs. 9 and 10. The contours, as previously discussed, are seen to give the form of a volcano whose crater has a rounded rim and a conical interior reaching to the epicenter.

Now let a number of parallel sections be taken at horizontal distances from the epicenter equal to $0.0D$, $0.5D$, etc. The curves for Fx for each section are shown in Fig. 11C and for Fy in Fig. 11D. The points of maximum value for each section are indicated in A, B, C, and D and through these points are drawn the curves which are loci of maxima. For Fy there is a single maximum for each section and this is situated at $x=0$. For Fx there are in each section two equal maxima but of opposite sign, one for a plus, the other for an equal minus value of x. In Fig. 11C only one side of the curve is shown, that for plus values of x.

Fig. 11B shows the points giving maximum values of Fx projected onto the vertical plane passing through X–X. The dip angle θ measures the slope from the point of maximum value to the center of mass. Here is shown its value when projected onto the plane X–X. This projected angle is seen to grow smaller with each more eccentric position of the section plane.

For section 0.0D the real value of θ is 55°
For section 1.0D the projection of θ' is 45°
For section 2.0D the projection of θ'' is 32°
For section 3.0D the projection of θ''' is 24°

Therefore it is seen that if the traverse line were assumed to pass through the epicenter of all disturbing masses, the error introduced would be to show the center of mass deeper than it really is. The nature, however, of the geodetic data permits this assumption to be eliminated and the distance EE' to the section plane to be approximately determined. An error up to 0.5D will not involve much error in the resultant depth as determined by the projection of θ. At each station along a line of triangulation[1] both Fx and Fy are determined and their resultant points toward the center of gravitative control. Each station gives an independent determination of this resultant and the intersection of two resultants if accurately determined and due to the gravitative force of a single symmetrical mass would give an accurate location of the epicenter, measuring its distance and direction from the traverse line. The data in many cases permit as many as three or four resultants to be drawn, the size of the triangle of their mutual intersections showing to what degree the forces may be ascribed to a single center. The relative positions of the line of section and epicenter of mass are thus in many cases approximately established.

But although the relative position of epicenters and traverse line are thus ascertained, the depth of the masses remains to be solved. In Fig. 11 the distance of the traverse line from the epicenter is given in terms of D, but this is the unknown. Two independent methods lead up to the solution of D.

First, on any line of section occurs a zero point for Fx. Let this be called E'. On each side of the zero point for Fx occurs a maximum value for Fx. Let the point of this maximum be called M. These two values are given by the geodetic data. Then for any spherical mass the ratio of $\frac{EE'}{E'M}$ increases with increase in the ratio of $\frac{EE'}{D}$ but not as a rectilinear function. This relation of ratios is shown graphically in Fig. 11E, in which the abscissas are the values of $\frac{EE'}{D}$ and the ordinates are the corresponding ratios of $\frac{EE'}{E'M}$. This ratio may be determined from the geodetic data but *from the location of the maxima, not their amount.*

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[1] As shown in illustration No. 3, Hayford, ***Supplementary Paper.***

The second method for determining the depth depends upon the ratio of the maximum value of Fy for any traverse line to the maximum value of Fx for the same traverse line, thus being dependent upon the *relative values of the maxima and not their location.* This ratio also increases with increase in the ratio of $\frac{EE'}{D}$ but not as a rectilinear function. The maximum Fx for a section at any distance from E is shown in Fig. 11C. For example, if $E'E=0.5D$ the maximum Fx for the unit sphere is $5.52''$. The maximum Fy is shown in Fig. 11D, and for $E'E=0.5D$ is $6.42''$. The ratio of 5.52 to 6.42 is 1.16. These ratios are shown in Fig. 11F for all traverse lines up to a distance of $3.0D$ from the epicenter. The value of this ratio is given by the geodetic data for any traverse line and hence the distance to the epicenter is given in terms of D.

In conclusion on this topic it may be said that the curves shown in Figs. 11E and 11F are independent of the mass or volume of the sphere, depending only upon its depth, and are adapted to use with the geodetic data. It is seen from both curves that the significant ratios change in value rapidly with increasing distance of the traverse line from epicenter up to a distance $E'E=D$, but beyond this point the change in the value of the ratios becomes progressively small as compared to a change in the distance of the section plane. The method is therefore well adapted for determining the depth of the outstanding masses assumed as spheres, provided the section line is not farther from the epicenter than the latter is above the center of mass. The method may be used, however, with less precision for distances of the traverse line *E′E* up to 2*D*. Beyond this distance, however, influences of other masses or errors in the geodetic data would be likely to give wholly erroneous results, not distinguishing between a large excess of mass at a great depth or a smaller one at a much less depth.

[*To be continued*]